Practice §12.4: Cross Product

Instructions: Complete each of the following exercises for practice.

1. Compute the cross product $\mathbf{u} \times \mathbf{v}$ and verify it is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.

   (a) $\mathbf{u} = \langle 4, 3, -2 \rangle$, $\mathbf{v} = \langle 2, -1, -1 \rangle$

   (b) $\mathbf{u} = 2\mathbf{j} - 4\mathbf{k}$, $\mathbf{v} = -\mathbf{i} + 3\mathbf{j} + \mathbf{k}$

2. State whether each expression is meaningful or meaningless. Explain.

   (a) $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$

   (b) $\mathbf{u} \times (\mathbf{v} \cdot \mathbf{w})$

   (c) $\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$

   (d) $\mathbf{u} \cdot (\mathbf{v} \cdot \mathbf{w})$

   (e) $(\mathbf{u} \cdot \mathbf{v}) \times (\mathbf{w} \cdot \mathbf{x})$

   (f) $(\mathbf{u} \times \mathbf{v}) \cdot (\mathbf{w} \times \mathbf{x})$

3. Find two unit vectors orthogonal to both $\langle 3, 2, 1 \rangle$ and $\langle 1, 2, 3 \rangle$.

4. Find the area of the parallelogram with vertices $A = (-3, 0)$, $B = (-1, 3)$, $C = (5, 2)$, and $D = (3, -1)$.

5. Find a nonzero vector orthogonal to the plane containing $P = (1, 0, 1)$, $Q = (-2, 1, 3)$, and $R = (4, 2, 5)$.

6. Find the volume of the parallelopiped determined by the vectors $\mathbf{u} = \langle 1, 2, 3 \rangle$, $\mathbf{v} = \langle -1, 1, 2 \rangle$, and $\mathbf{w} = \langle 2, 1, 4 \rangle$.

7. Prove the properties of the cross product stated in class.